# UCLA
# Space Physics Exercises
# Manual

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# Overview

The UCLA Space Physics Exercises website contains a series of modules designed to introduce you to the following topics in space physics: magnetospheres, particle motion, plasma waves, magnetohydrodynamics (MHD)/shocks, solar wind, currents, and the ionosphere.

You can access the Space Physics Exercises website at the following URL:

https://spacephysics.ucla.edu/

The modules for each of the topics mentioned above are organized into separate screens that you can access using the row of tabs at the top of the home screen:


When you click on one of the tabs, the row of tabs will change. If you see only one tab besides the **HOME** tab, then there is only one module for that topic. If you see more than one tab, those tabs allow you to access all of the modules for that topic.

No matter where you are in the Space Physics Exercises website, you can always click the **HOME** tab to get back to where you started.

## Textbook that uses these exercises

You can find sets of problems that use these exercises in the Cambridge University Press textbook *Space Physics: An Introduction* by C. T. Russell, J. G. Luhmann, and R. J. Strangeway.

Please visit the publisher's web page www.cambridge.org/spacephysics for more information.

# Currents

The **Currents** topic consists of two modules, **Auroral Electrojet** and **Dst Case Studies**. You can switch between these two modules using the row of tabs along the top of the page:


As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let's go over the two modules of the **Currents** topic.

## AURORAL ELECTROJET

The **Auroral Electrojet** module illustrates how ionospheric and magnetospheric electrojets cause the signatures seen on the ground beneath them and how the solar wind and interplanetary magnetic field can lead to the buildup of the ring current.

The module initially looks like this:

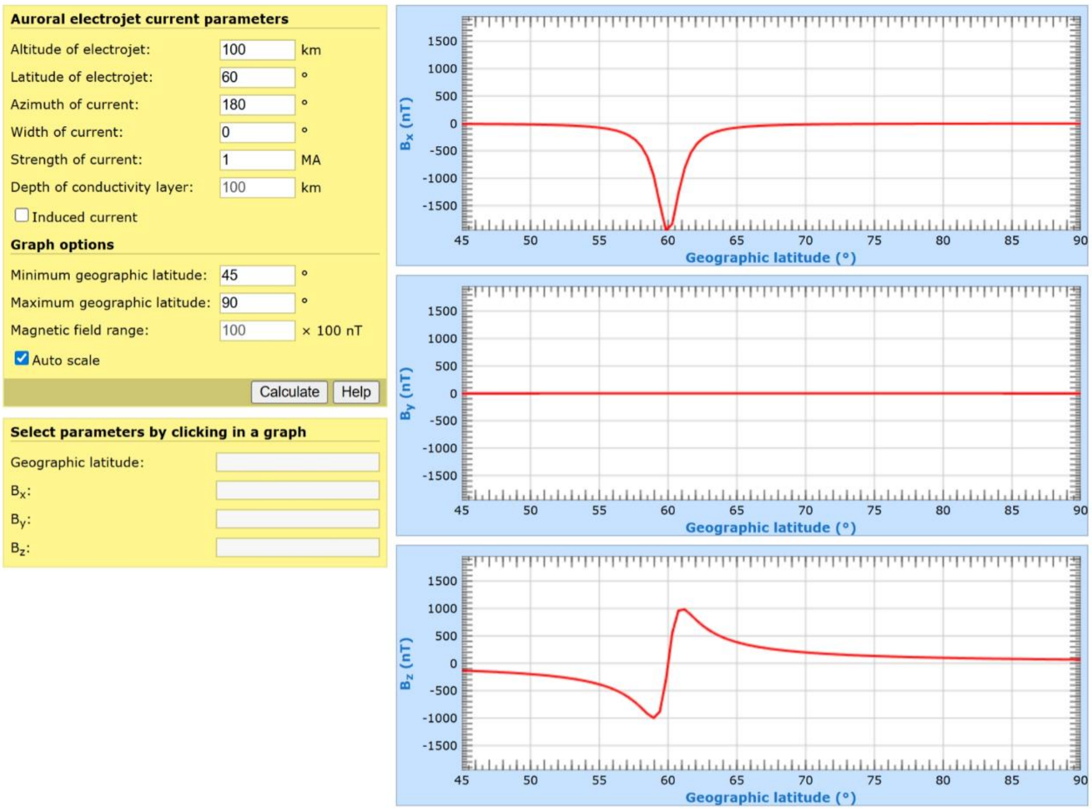



**Figure 1:** The **Auroral Electrojet** module.

The **Auroral Electrojet** module allows you to determine the magnetic field at the surface of the Earth due to an infinite line current (or multiple line currents) in the ionosphere above. Such a narrow "line" in the ionosphere is known as an electrojet. The surface of the Earth is taken to be flat, and the Earth itself to be insulating, but an infinitely conducting layer can be placed at any depth from 0 to 200 km. The currents in this sheet may be turned on and off using the **Induced current** check box.

The location of the observer, the orientation of the current flow, and the components of the magnetic field are all specified in geographic coordinates in which X is north, Y is east, and Z is down. This is the traditional coordinate system used in geomagnetism. An alternative that is sometimes used is the HDZ coordinate system, where Z is the same as in the XYZ system, H is along the projection of the average magnetic field on the surface of the Earth, and D is perpendicular to H and Z, and is roughly eastward. Originally, the D coordinate was an angle measuring the direction of the horizontal projection of the field away from the average direction.

The current flows in the horizontal plane above the observer at an altitude of 50 to 150 km. The latitude of the center of the current is adjustable from 0° to 90°. Since geomagnetic phenomena are usually aligned in some direction other than strictly east-west geographic, the orientation of the current in the horizontal plane (the azimuth) can be rotated. For a 0° azimuth the current flows from west to east geographic. For a 180° azimuth the current flows from east to west. This is the direction of the usual westward electrojet due to substorms. A 90° azimuth current flows due south and a -90° azimuth current flows due north.

The width of the current can be altered keeping the current centered at its specified location. The program places a wire every 0.1 over the interval specified and splits the total current evenly among these wires. The strength of the current can be specified in steps of 0.1 MA from 0 to 10 MA.

The default mode of the program is to have no currents induced in the Earth. However, an infinitely conducting layer may be switched on. This layer shields the interior of the Earth from the magnetic field produced by the ionospheric current. This shielding is equivalent to the introduction of a mirror current flowing parallel to the ionospheric current at a depth below the conducting layer equal to the sum of the depth of the conducting layer and the height of the ionospheric current. If the ionospheric current is narrow and the conducting sheet is at the surface of the Earth, the X component of the field at the surface of the Earth is doubled and the Z component goes to zero everywhere for an east-west current. If the sheet is at a depth of 200 km so that the mirror current is at 400 km, there is a small enhancement of the X and Z components over the non-conducting sheet case. For a broad sheet of current flowing east-west the conducting sheet will double the north-south field if it is at the surface and increase it 50% if at 200 km depth.

The north-south extent of the plot can be changed by specifying the minimum and maximum geographic latitudes. To measure the three components of the field at any latitude, move the pointer to that latitude and click the mouse. The measurement will appear in the boxes to the left. Click **Calculate** to redraw the graphs after changing any of the parameters. The vertical scale of the three graphs is kept the same so that the variations of the three components will be displayed in their proper relative proportions.

## DST CASE STUDIES

The **Dst Case Studies** module illustrates how ionospheric and magnetospheric electrojets cause the signatures seen on the ground beneath them and how the solar wind and interplanetary magnetic field can lead to the buildup of the ring current.

The module initially looks like this:


**Figure 2:** The **Dst Case Studies** module.

The **Dst Case Studies** module illustrates how Dst, the disturbance storm time index, may be predicted from a knowledge of solar wind parameters. The formula used is that of Burton *et al*. ("An empirical relationship between interplanetary conditions and Dst," *J. Geophys. Res.*, 80, 4204–4214, 1975). You can change the parameters in the Burton formula and compare the new prediction with the observed values for a specific event.

**Dst**

A measure of the worldwide deviation of the H component of the Earth's magnetic field at mid-latitude ground stations from their quiet day values. It is measured in nanoTeslas (nT).

Dst is composed of perturbations due to the ring current ($Dst_0$), the magnetopause currents ($bP_d^{1/2}$), and the quiet day currents (c).

$$Dst = Dst_0 + bP_d^{1/2} - c$$

**$Dst_0$**

The measure of the perturbation of the H component due to the ring current. Essentially, it is a measure of the strength of the ring current. $Dst_0$ is measured in nanoTeslas (nT).

The ring current changes strength as a result of energy injection and decay.

$$d(Dst_0) / dt = F(E) - aDst_0$$

**a**

The decay rate of the ring current, $3.6 \times 10^{-5}$ $s^{-1}$. 1/a is the decay time of the ring current, 7.7 hours. This parameter can be varied from 1–20 hours using the **Ring current decay time, a** box.

**b**

The measure of the response in Dst to changes in the solar wind dynamic pressure ($P_d = mnV^2$, where m is the ion mass, n is the ion density, and V is the solar wind velocity). It is 15.8 $nT/nPa^{1/2}$. This parameter can be varied from 5–40 $nT/nPa^{1/2}$ using the **Response pressure, b** box.

**c**

The measure of the quiet day currents, that is, the currents that exist when the solar wind dynamic pressure is nearly constant and F(E) = 0 (i.e., when $B_z$ is northward). It is 20 nT. This parameter can be varied from 0–50 hours using the **Quiet day ring current, c** box.

**F(E)**

Ring current injection parameter F(E) is the rate of ring current injection as a function of a rectified, filtered, and delayed duskward component of the solar wind electric field ($E_y$). This parameter can be varied from 0–60 minutes using the Ring current injection rate, F(E) box. The only filtering in this module is the 5-minute averaging performed in creating the input data files.

$$F(E) = d(E_y - 0.5) \text{ if } E_y > 0.5 \text{ mV/m}$$

$$F(E) = 0 \text{ if } E_y < 0.5 \text{ mV/m}$$

$E_y = -VB_z$, where V is the solar wind velocity and $B_z$ is the north/south component of the interplanetary magnetic field (IMF).

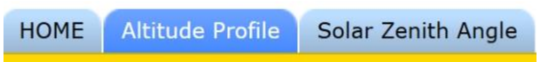
d is the change in injection rate as a function of the rectified and filtered solar wind electric field. It is $1.5 \times 10^{-3}$ nT/(mV/m)s.

Rectified means that energy is transferred into the magnetosphere only if $E_y > 0$. This occurs when $B_z$ is southward. Experience (Burton et al., 1975) shows that the magnetosphere acts as a low-pass filter: quickly varying electric fields are not as well rectified as slowly varying electric fields. By averaging to 5-minute resolution we have accounted for some of the effect of the magnetospheric filter but perhaps not all of it. The response in F(E) is delayed with respect to changes in solar wind $E_y$ by the delay time $t_m$. $t_m$ is the average time delay between changes in the solar wind electric field observed at the nose of the magnetopause and observed ring current injection (i.e., decreases in Dst).

The values in the graphs may be read by moving the pointer over a graph and clicking the mouse. The readings will appear in the boxes to the left.

## IONOSPHERE

The **Ionosphere** topic consists of two modules, **Altitude Profile** and **Solar Zenith Angle**. You can switch between these two modules using the row of tabs along the top of the page:



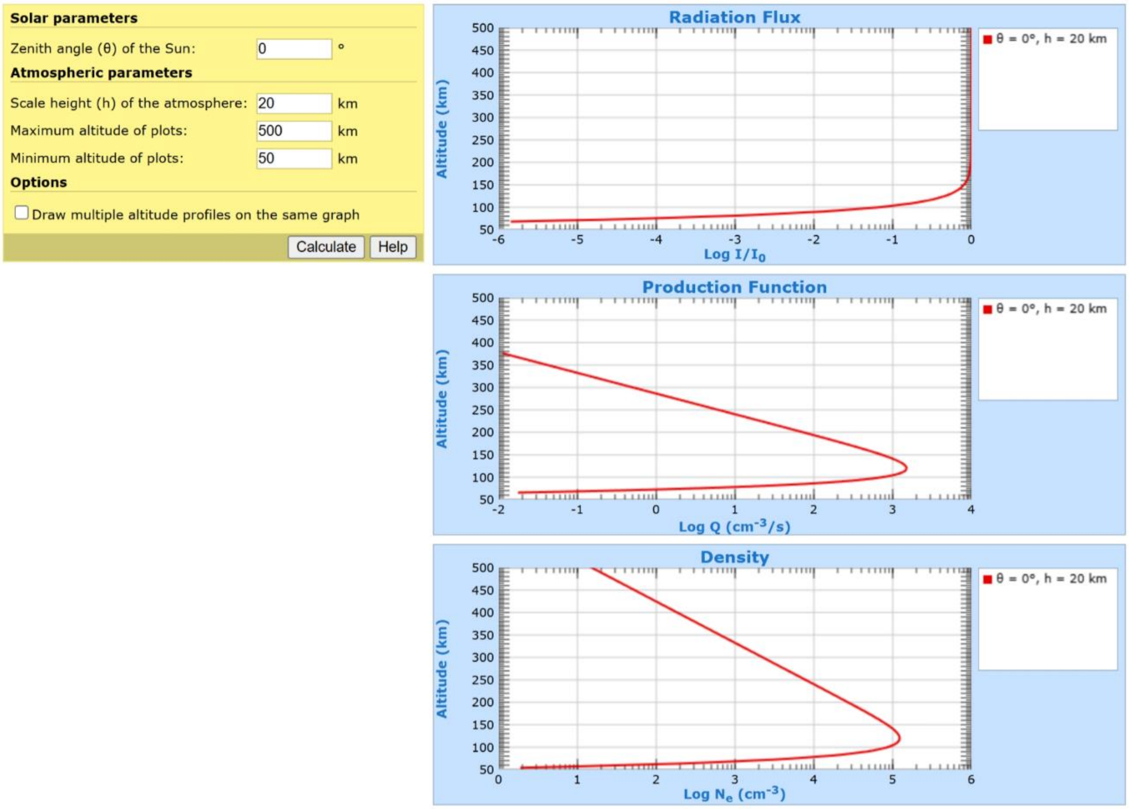

As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let’s go over the two modules of the **Ionosphere** topic.

### ALTITUDE PROFILE

The **Altitude Profile** module allows you to plot altitude profiles of the radiation flux from the Sun as it is absorbed coming into the atmosphere, the number of electron-ion pairs produced, and the resulting density in an α-Chapman layer where the recombination rate is proportional to the square of the electron number density.

The module initially looks like this:




We assume the Chapman production function peaks at 120 km and that at 0° solar zenith angle and with an atmospheric scale height of 10 km the production function is $8 \times 10^3$ electrons $cm^{-3}$ $s^{-1}$.

**Figure 3:** The **Altitude Profile** module.

The ionosphere is produced above about 90 km altitude by two principal sources of atmospheric ionization: photoionization by solar radiation and energetic particle "impact" ionization. Photoionization is primarily confined to the day-side hemisphere, although there is some sunlight scattered over the terminator (the day-night boundary), and on unmagnetized planets pressure gradient forces can cause night-ward flow of ionospheric ions produced on the dayside.

To ionize an atmospheric atom or molecule, solar photons must have energies at least equal to the binding energy of the electron. For most atmospheric gases, the necessary energies fall in the extreme ultraviolet (EUV) wavelength range rather than in the visible range. This wavelength range exhibits average flux variations of about a factor of two to three during the solar cycle, causing solar cycle variations in the dayside ionosphere.

To calculate the ionization produced by a flux of photons, one must first determine the solution to a radiative transport equation. This solution describes the attenuation with depth of the incident photon flux given knowledge of the cross sections for photon absorption. Years ago, Chapman studied the simple case of monochromatic radiation incident on a single constituent atmosphere having constant scale height h (the scale length for exponential fall-off of the atmospheric density with increasing height). The solutions for the resulting profiles of photon energy deposition and hence ion production Q (assuming one ion pair per 35 eV deposited) are the basis of the Chapman layer model of an ionosphere. Electron density profiles are derived from the production rates by assuming that recombination at a rate proportional to $n^2$ is the only loss process, whereby n is proportional to $Q^{1/2}$.

The Chapman layer model, which is described in most basic texts on ionospheric physics (e.g. Rishbeth and Garriott; Ratcliffe) can apply to radiation incident at any angle from the zenith, and hence can be used to compare ionosphere properties at different times of day and different latitudes. The constant neutral atmosphere scale height can also be varied to emulate the effects of changing solar activity conditions or the different gravitational fields, temperatures, and compositions at other planets. Here we have programmed the standard Chapman layer model in a way that allows you to experiment with both these variables.

On Earth the atmospheric scale height varies from about 5 km at 80 km altitude to 80 km at 500 km altitude. On Venus the carbon dioxide atmosphere and cold temperatures of the nighttime upper atmosphere can result in scale heights as low as 2 km.

The equations of Chapman layer theory will not be repeated here, given their accessibility in the literature.

## Solar Zenith Angle

The **Solar Zenith Angle** module allows you to see how the density and height of the electron density maximum varies with the solar zenith angle.

The module initially looks like this:

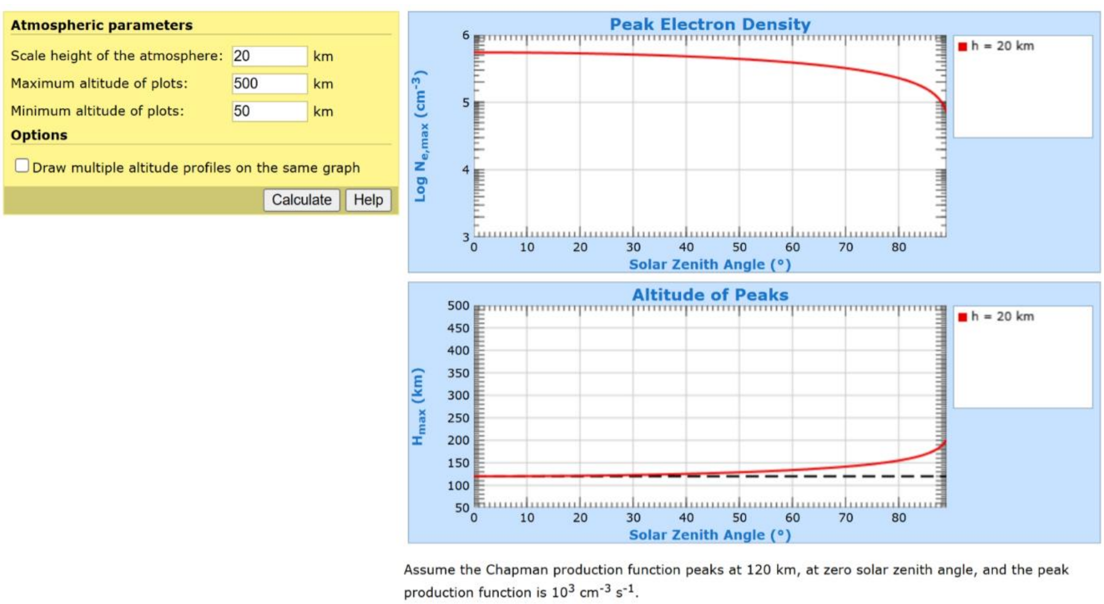



**Figure 4:** The **Solar Zenith Angle** module.

The **Solar Zenith Angle** module allows you to plot the logarithm of the peak electron density and the altitude of the peak electron density versus solar zenith angle.

For additional information on Chapman layer production, see the description of the **Altitude Profile** module above.

## MAGNETOSPHERES

The **Magnetospheres** topic consists of only a single module, also called **Magnetospheres**.


As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let's go over the single module of the **Magnetospheres** topic.

### MAGNETOSPHERES

The **Magnetospheres** module is designed to introduce you to some of the elementary properties of planetary magnetic fields.

The module initially looks like this:

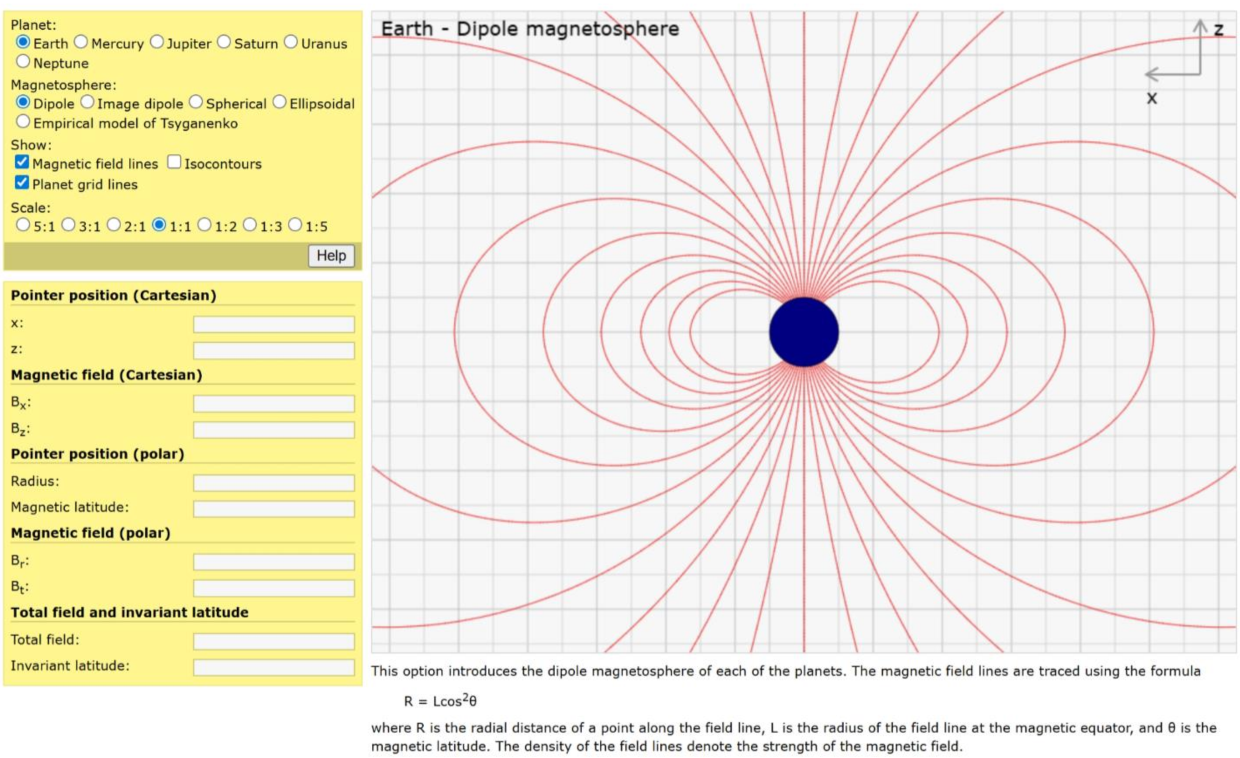



**Figure 5:** The **Magnetospheres** module.

**Planet**

You can select to display the magnetic fields of the planets Earth, Mercury, Jupiter, Saturn, Uranus, and Neptune. The planets Venus and Mars have no intrinsic magnetic field detectable by orbiting spacecraft. There are currently no plans to measure the magnetic field of the dwarf planet Pluto.

**Magnetosphere**

You can choose among the following magnetic field options: dipole, image dipole, spherical, elliptical, and the empirical model of Tsyganenko. These options are described below.

- **Dipole**

  This option shows the dipole magnetic field lines, the directions that one would follow with a three-dimensional compass for each of the planets that has a significant planetary magnetic field. The size of the region in which the magnetic field is drawn is proportional to the size of the magnetic cavity, or magnetosphere that the planet carves out of the solar wind. Thus, tiny Mercury with its very weak magnetic field appears large because its magnetosphere is small, while the planet Jupiter appears very small because in comparison its magnetosphere is very large.

  For these displays we have kept the magnetic axis of each planet vertical so that the rotation axis is tilted an amount equal to the angle between the magnetic dipole axis and the rotation axis. On Saturn this angle is zero, while on the Earth it is 11°. A dipole field is the simplest approximation to a planetary magnetic field. Actual planetary magnetic fields can be much more complex than this, especially near the planet's surface.

  The dipole magnetic field in a vacuum is the simplest magnetic configuration encountered in nature. It arises due to currents in a loop. The strength of such a current loop is called its magnetic moment and is proportional to the area of the loop times the current carried by the loop. The dipole magnetic moment is also numerically equal to the value of the magnetic field at any point in the equatorial plane times the cube of the distance from the center of the body. The magnetic moment of the Earth is approximately $8 \times 10^{15}$ $Tm^3$.

- **Image dipole**

  The magnetized plasma of the solar wind blows against the intrinsic magnetic fields of the planets, confining the planetary field to a cavity called the magnetosphere. In this and the other displays the solar wind blows from the left, confining the planetary field and stretching it out to the right. The lines emanating from each planet are called lines of magnetic force or magnetic field lines. They are displayed in the noon-midnight meridian.

  The magnetosphere is a complex and dynamic system. In order to understand this system this module presents a series of approximations to the Earth's magnetosphere of increasing complexity. The first model that displays some of the properties of a real magnetosphere is the image dipole model displayed here. The image dipole model was the first model of the Earth's magnetosphere and it was proposed by S. Chapman and V.C.A. Ferraro in 1931. They correctly hypothesized that the solar wind was highly electrically conducting and would exclude the Earth's field from its interior, compressing the field to the right-hand side of this diagram. If the interface with the solar wind were a flat plane, the distortion in the magnetic field on the right-hand side of the plane would be the same as if there were a dipole field on the left-hand side of the plane at the same distance from it and of the same strength (magnetic moment). This is equivalent to the Earth's magnetic field being reflected in a mirror and is therefore called the image dipole model of the magnetosphere.

The magnetic field at the equator at the forward boundary of the magnetosphere, the interface with the solar wind is exactly double that of a pure dipole at the same location. The field component normal to the interface is zero. There are two null points where the total field is zero and above and below which the field points in opposite directions parallel to the plane. Bringing the image dipole closer to the Earth simulates the effect of an increase of the solar wind pressure. This model was developed by Chapman and Ferraro to explain the phenomenon called the geomagnetic storm.

- **Spherical**

  The interface between the solar wind and the magnetic field is not a flat plane but a curved surface. To examine the effect of the curvature of this surface on the magnetic field inside the magnetosphere, this option shows the situation in which the Earth's dipole finds itself in a spherical superconductor. This resembles somewhat the situation in which the solar wind velocity dropped to zero but it retained sufficient thermal pressure (the extended solar corona) to confine the magnetosphere on all sides. It differs from this case because the pressure required to confine the magnetic field of a dipole within a spherical surface is not uniform over that surface.

  As shown here the field is compressed and enhanced everywhere, but most noticeably in the outer reaches of the magnetosphere near the equator, where it is tripled in strength. This compares with the doubling of the field strength at the boundary, or magnetopause, in the case of the image dipole model.

- **Ellipsoidal**

  The ionized, extended, upper atmosphere of the Sun, called the solar corona, is not static but expands outward from the Sun. This confines the Earth's magnetic field on the sunward, or dayside, of the magnetosphere, and stretches it out in the antisolar direction.

  On the dayside of the Earth the magnetopause can be approximated by an ellipsoid of revolution symmetric about the Earth-Sun line with the Earth at one focus of the ellipsoid. This option lets us examine the magnetic structure of such a magnetosphere as first developed by N. Tsyganenko in 1989. The figure shows the magnetic field in the noon-midnight meridian plane for a dipole magnetic field enclosed by a ellipsoidal conductor. The nose of the magnetosphere is 10.0 $R_{Earth}$ in front of the Earth and the end of the tail is 76.6 $R_{Earth}$ behind the Earth. In this model the dayside magnetic field is more compressed than in the image dipole model but less than in the spherical model. The nightside field is weaker than in the image dipole and spherical models but not as weak as in the real magnetosphere, in which the plasma provides some of the pressure that balances the pressure applied by the solar wind.

- **Empirical model of Tsyganenko**

  The image dipole, spherical, and ellipsoidal magnetosphere options present the expected field configuration of a magnetosphere devoid of plasma. Thus, no electric currents could flow within the magnetosphere, either along or across the magnetic fields lines. The only electric currents that could flow were within the Earth and on the surface of the magnetosphere (i.e., the magnetopause). This option shows the model developed by Tsyganenko (1989) based on magnetic field observations that includes currents within the magnetosphere, and simulates more realistically the actual field configuration of the Earth's magnetosphere. The figure shows the magnetic field in the noon-midnight meridian plane for the empirical model of Tsyganenko.

**Show**

- **Magnetic field lines**

  Shows the planet's magnetic field lines. A *magnetic field line* is a path through a magnetic field that is everywhere parallel to the local magnetic field.

- **Isocontours**

  Shows the contours along which magnetic field strength is constant.

  For the image dipole magnetosphere, the solar wind acts to force the contours slightly closer to the Earth on the sunlit side (left) as opposed to the nightside (right). This difference in strength between noon and midnight can be observed (on the surface of the Earth) as a diurnal variation.

  Because of the asymmetry of the magnetic field in the ellipsoidal and empirical models, a diurnal change is seen as the Earth turns. Because of the presence of plasma in the tail and the cross tail current, the diurnal variation is greater for the empirical model than other models in this module.

- **Planet grid lines**

  Displays a grid, with spacing between lines equal to the displayed planet's radius.

### MEASURING THE MAGNETIC FIELD USING THE MOUSE

To measure the planetary magnetic field, move the pointer across the screen and click the mouse. A small random component has been added to the location of the cursor so that repeated clicks at apparently the same location are in fact at slightly different locations and result in different measured fields. This overcomes limitations of the finite digitization of locations on the screen.

The coordinate system in which the measurements are made and written in the boxes on the left are aligned with the magnetic dipole axis (z) and the equatorial plane (x). The coordinate z-axis is upward in the diagram whether or not the magnetic dipole is aligned parallel or antiparallel to this direction. The coordinate x-axis is positive to the left. The origin of the coordinate system is the center of the planet.

The quantities in the boxes are as follows.

**Pointer position (Cartesian)**

Gives the x and z location of the measurement in planetary radii.

**Magnetic field (Cartesian)**

Gives the two components of the magnetic field in nanoTeslas (nT). A Tesla is a $Wb/m^2$ or 10,000 Gauss.

**Pointer position (Polar)**

Gives the position of the pointer in polar coordinates. The **Radius** box gives the distance from the center of the planet in planetary radii. The **Magnetic latitude** box gives the angle in degrees that the radius vector makes with the equator, i.e., the latitude of the point of observation. The *colatitude* is 90° - θ.

**Magnetic field (Polar)**

Gives the two components of the magnetic field in a polar coordinate system. The $B_R$ component is radially outward. The $B_T$ component is tangential to the surface of the planet and in the direction of the colatitude, i.e., from the upward pole toward the equator. It is downward at the Earth's equator.

**Total field and invariant latitude**

Gives the total magnetic field, which is the square root of the sum of the squares of $B_R$ and $B_T$, and the invariant latitude, which is the magnetic latitude in degrees where the field line meets the surface of the planet. If L is the equatorial distance from the center of the planet to the field line, then the invariant latitude is the angle whose cosine is the reciprocal of the square root of L.

## MHD/SHOCKS

The **MHD/Shocks** topic consists of five modules, **Rankine-Hugoniot Graphs**, **Rankine-Hugoniot Case Studies**, **MHD Wave Graphs**, **MHD Wave Case Studies**, and **Specularly Reflected Ions**. You can switch between these five modules using the row of tabs along the top of the page:


As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let's go over the five modules of the **MHD/Shocks** topic.

### RANKINE-HUGONIOT GRAPHS

The **Rankine-Hugionoit Graphs** module allows you to vary the Mach number that measures the strength of the shock, the plasma β that measures the ratio of the thermal to magnetic pressure, the angle between the upstream field and the shock normal, and the polytropic index in the ideal gas law, and then to view graphs of the jump in magnetic field strength, number density, temperature, and plasma β. Select **None** to calculate discrete values for the jumps in the plasma quantities.

The module initially looks like this:

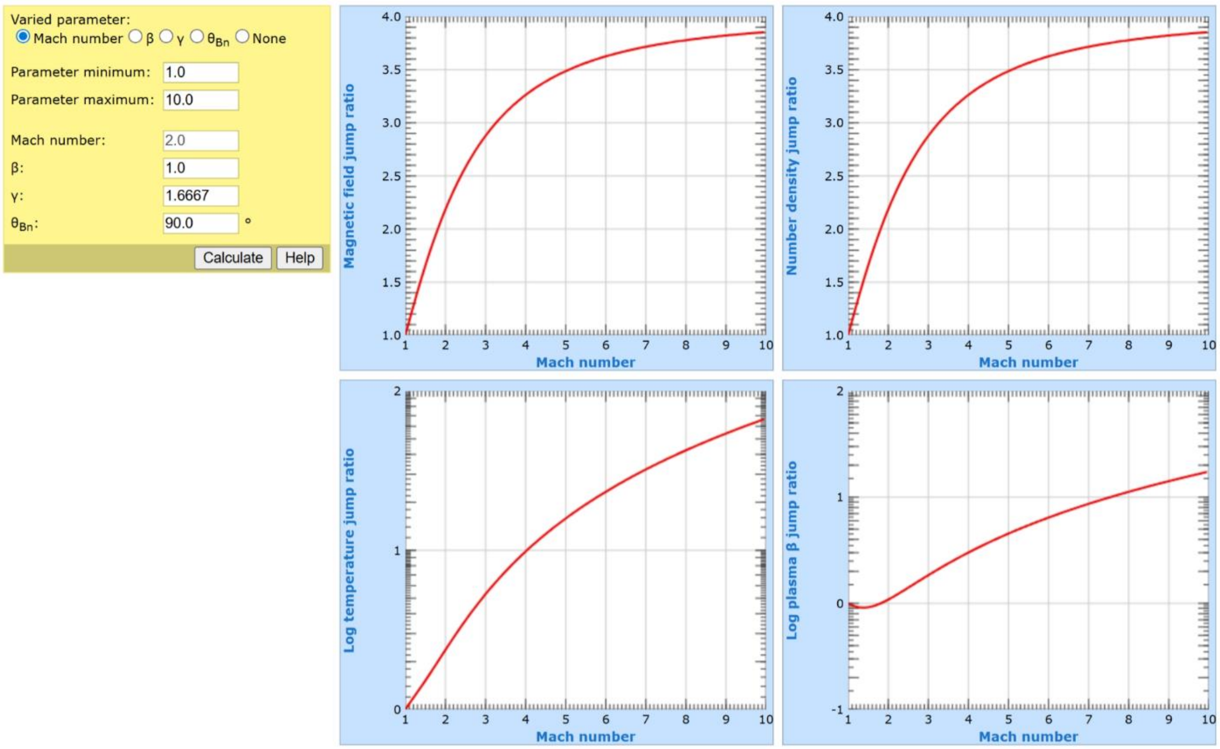



**Figure 6:** The **Rankine-Hugoniot Graphs** module.

The Rankine-Hugoniot equations are a set of conservation relations based on the equations of motion and Maxwell's equations that govern the changes of a medium that has passed through a shock wave. This module solves for the properties of the plasma downstream of the shock, based on the properties of the upstream plasma. Here the plasma is assumed to be isotropic and Maxwellian both upstream and down. The downstream state can vary based on four main properties of the upstream solar wind plasma. These parameters are:

**Mach number**

The ratio between the upstream flow velocity along the shock normal direction and the velocity at which fast magnetosonic compressional waves propagate. This parameter ranges from 1 to ∞.

**β**

The ratio between the thermal pressure and the magnetic pressure of the plasma. This parameter ranges from 0 (magnetic pressure dominates) to ∞ (thermal pressure dominates).

**γ**

The ratio of specific heats of the plasma. This parameter describes the number of degrees of freedom of the plasma, and generally ranges from 1 (infinite degrees of freedom) to 3 (one degree of freedom).

**$\theta_{Bn}$**

The angle between the magnetic field and shock normal directions. This parameter ranges from 0° (parallel orientation) to 90° (perpendicular orientation). When $\theta_{Bn}$ is 90° the magnetic field is in the shock plane. When $\theta_{Bn}$ is 0° the magnetic field is along the shock normal.

In this module, you can determine how the jumps in the magnetic field strength, number density, temperature, and plasma β vary as these parameters vary. The jumps are defined as the ratio of the downstream value of the quantity to the upstream value. To see this, select one of the options **Mach number**, **β**, **γ**, **$\theta_{Bn}$**. Then enter the minimum and maximum values for the varied parameter; the parameter will be varied within the specified range. Enter the values for the other three parameters in the boxes, and then click **Calculate** to plot the graphs.

The upper two graphs have linear vertical scales. The lower two graphs have logarithmic vertical scales.

If you want to find the exact solution for a given set of upstream parameters, you can do this by selecting **None** for the varied parameter. When you enter values for the four upstream parameters and click **Calculate**, the exact values for the jumps in magnetic field strength, number density, temperature, and plasma β are displayed next to the input parameters.

The ratio of criticality is also calculated. The ratio of criticality indicates whether the shock is subcritical (< 1), marginally critical (~ 1), or supercritical (> 1). This ratio becomes important in describing the physical processes observed at collisionless shocks. It is defined as the Mach number at which the downstream sound speed matches the downstream bulk velocity.

If you want to calculate downstream parameters from the upstream parameters for a standing shock, then use the **Rankine-Hugoniot Case Studies** module described below.

## Rankine-Hugoniot Case Studies

The **Rankine-Hugioniot Case Studies** module allows you to vary the upstream parameters for the plasma, such as velocity, magnetic field strength, temperature, number density, and so on, and then to calculate the downstream values for these quantities.

The module initially looks like this:

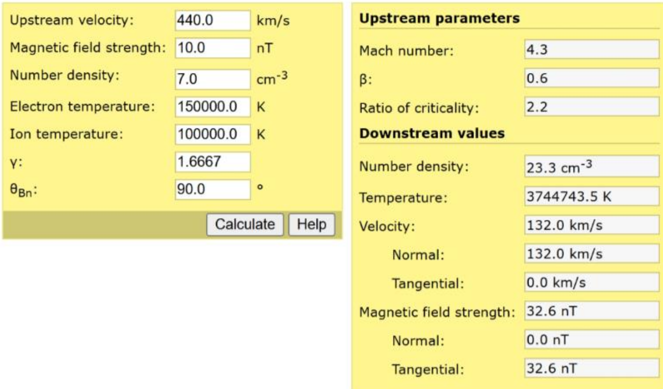



**Figure 7:** The **Rankine-Hugoniot Case Studies** module.

For introductory comments on the Rankine-Hugoniot relations, and definitions, see the description of the **Rankine-Hugoniot Graphs** module above.

The upstream parameters can range all the way to infinity. If you want to use infinity as a parameter value, type a very large number, like 10000, instead. This will yield results differing little from the solution for infinity.

The upstream velocity is measured along the shock normal, and two temperatures are used as inputs to remind you that both temperatures are relevant. The module calculates only a single downstream temperature.

## MHD Wave Graphs

The **MHD Wave Graphs** module allows you to see the phase velocity and group velocity of the fast, intermediate, and slow modes versus the angle of the field.

The module initially looks like this:

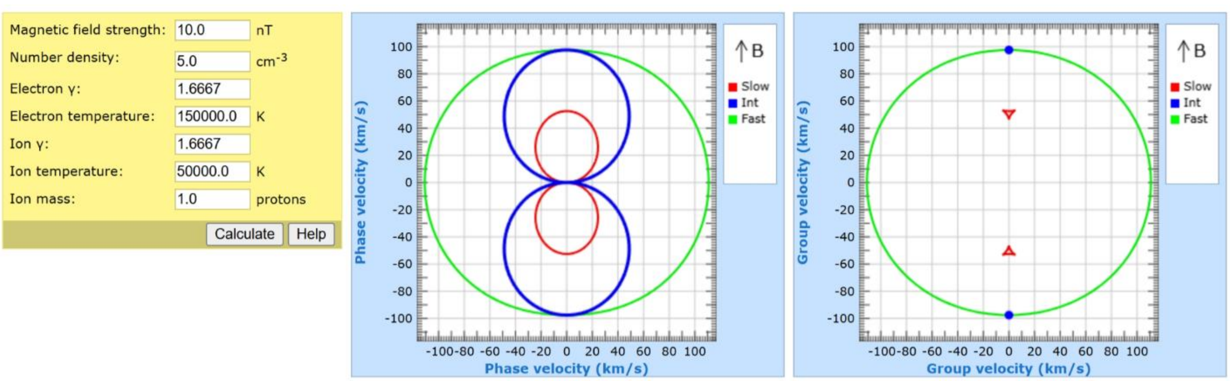



**Figure 8:** The **MHD Wave Graphs** module.

Magnetohydrodynamic (MHD) waves, are low-frequency waves (f ≪ ion gyrofrequency) in the plasma. In this module, we examine small amplitude waves propagating in a homogeneous, isotropic, and uniformly magnetized plasma. The program solves for the phase velocity and group velocity of the waves based on the properties of the plasma. The velocities for three MHD wave modes, fast, intermediate, and slow, are calculated by the program. A fourth mode is sometimes discussed, called the entropy fluctuation. This mode does not propagate.

Enter the parameters of the plasma on the left-hand side of the window. The defaults correspond to typical values in the solar wind. These parameters include: the magnetic field strength in nT, number density of the plasma in $\#/cm^3$, electron and ion γ (the ratio of specific heats) and temperature in K, and the ion mass in units of the mass of a proton. The electrons and ions are handled separately to allow different ratios of specific heat in the calculation of the sound speed. Click **Calculate** to update the graphs.

The left graph shows the wave phase velocities of the three MHD modes in km/s. The magnetic field direction is vertical. The polar plot shows the propagation velocities of the phase fronts of the three modes and their dependence on the angle between k (the wave vector) and B (the magnetic field).

The right graph shows the group velocities of the three MHD modes in km/s. The magnetic field direction is also vertical. The plot shows the direction of the energy propagation relative to the magnetic field. Thus, it shows how strictly the energy is constrained to follow the magnetic field. It should not be interpreted as the group velocity dependence on the direction of the phase velocity to the magnetic field, which may be quite different. To explore this, use the **MHD Wave Case Studies** module described below.

## MHD Wave Case Studies

The **MHD Wave Case Studies** module allows you to specify the set of plasma conditions and the direction of propagation, and then to calculate the values of the phase and group velocities.

The module initially looks like this:

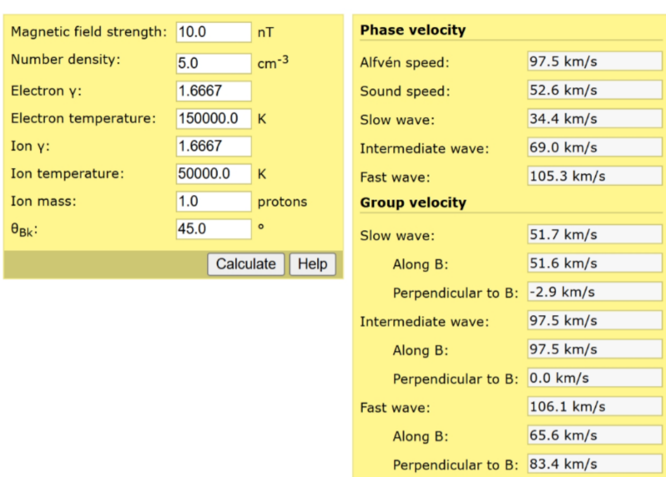



**Figure 9:** The **MHD Wave Case Studies** module.

Magnetohydrodynamic (MHD) waves, are low-frequency waves ($f \ll$ ion gyrofrequency) in the plasma. In this module, we examine small amplitude waves propagating in a homogeneous, isotropic, and uniformly magnetized plasma. Unlike the shock wave treated elsewhere, in this module the wave does not alter the properties of the medium through which it passes. The program solves for the phase velocity and group velocity of the waves based on the properties of the plasma. The velocities for three MHD wave modes, fast, intermediate, and slow, are calculated by the program. A fourth mode is sometimes discussed, called the entropy fluctuation. This mode does not propagate.

Enter the parameters of the plasma on the left-hand side of the window. The defaults correspond to typical values in the solar wind. These parameters include: the magnetic field strength in nT, number density of the plasma in #/cm$^3$, electron and ion $\gamma$ (the ratio of specific heats) and temperature in K, and the ion mass in units of the mass of a proton. The electrons and ions are handled separately to allow different ratios of specific heat in the calculation of the sound speed. Click **Calculate** to display phase velocities and group velocities of the three MHD wave modes in km/s. The parameter $\theta_{Bk}$ gives the direction of the magnetic field relative to the phase front of the wave. The group velocity need not propagate along the phase velocity. Thus, it must be specified by two components, here given as along the field and perpendicular to it.

## Specularly Reflected Ions

The **Specularly Reflected Ions** module allows you to vary the solar wind velocity, magnetic field strength, and angle of the field and shock normal, and then to calculate the distance to which ions are reflected back into the solar wind before they begin to drift back toward the shock.

The module initially looks like this:

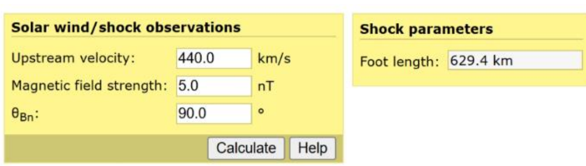



**Figure 10:** The **Specularly Reflected Ions** module.

The foot in the magnetic profile of quasi-perpendicular, supercritical shocks develops upstream of the main ramp when the magnetic field makes its main and irreversible jump to its downstream value. This foot is caused by gyrating ions which are nearly specularly reflected at the shock boundary, i.e., the incident ions' components of velocity parallel to the shock normal are reversed at the shock while components along the shock surface remain unchanged. These reflected ions are turned around by the upstream magnetic field and returned to the shock.

This module is based on the expression developed by J.T. Gosling and M.F. Thomsen ("Specularly reflected ions, shock foot thicknesses, and shock velocity determinations in space," *J. Geophys. Res.*, 90, 9893–9896, 1985) for the turnaround distance of specularly reflected ions at arbitrary orientations of incident velocity and upstream magnetic field.

Given this characteristic distance and the time taken for the shock foot to pass by the observing spacecraft, the shock velocity can also be determined (assuming that the velocity is constant throughout the foot observation).

These parameters are calculated from the following inputs:

**Upstream velocity**

The component of the incident velocity along the shock normal, in km/s.

**Magnetic field strength**

Measured in nT upstream of the shock.

**$\theta_{Bn}$**

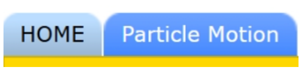
The angle between the magnetic field and shock normal directions. This parameter ranges from 0° (parallel orientation) to 90° (perpendicular orientation). When $\theta_{Bn}$ is 90° the magnetic field is in the shock plane. When $\theta_{Bn}$ is 0° the magnetic field is along the shock normal.

After entering the input parameters, click **Calculate** to display the foot length, the theoretical maximum distance from the shock reached by the ion if ions are perfectly specularly reflected at the shock boundary.

**Note:** A shock foot will form only for quasi-perpendicular shocks, because the reflected particles will escape upstream along the magnetic field for quasi-parallel shocks, and so results when $\theta_{Bn}$ is less than 39.9° when escape first occurs (for decreasing $\theta_{Bn}$) are not calculated.

## Particle Motion

The **Particle Motion** topic consists of only a single module, also called **Particle Motion**.




As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let's go over the single module of the **Particle Motion** topic.

### Particle Motion

The **Particle Motion** module allows you to follow the three-dimensional trajectories of charged particles and atoms through different magnetic field configurations.

The module initially looks like this:

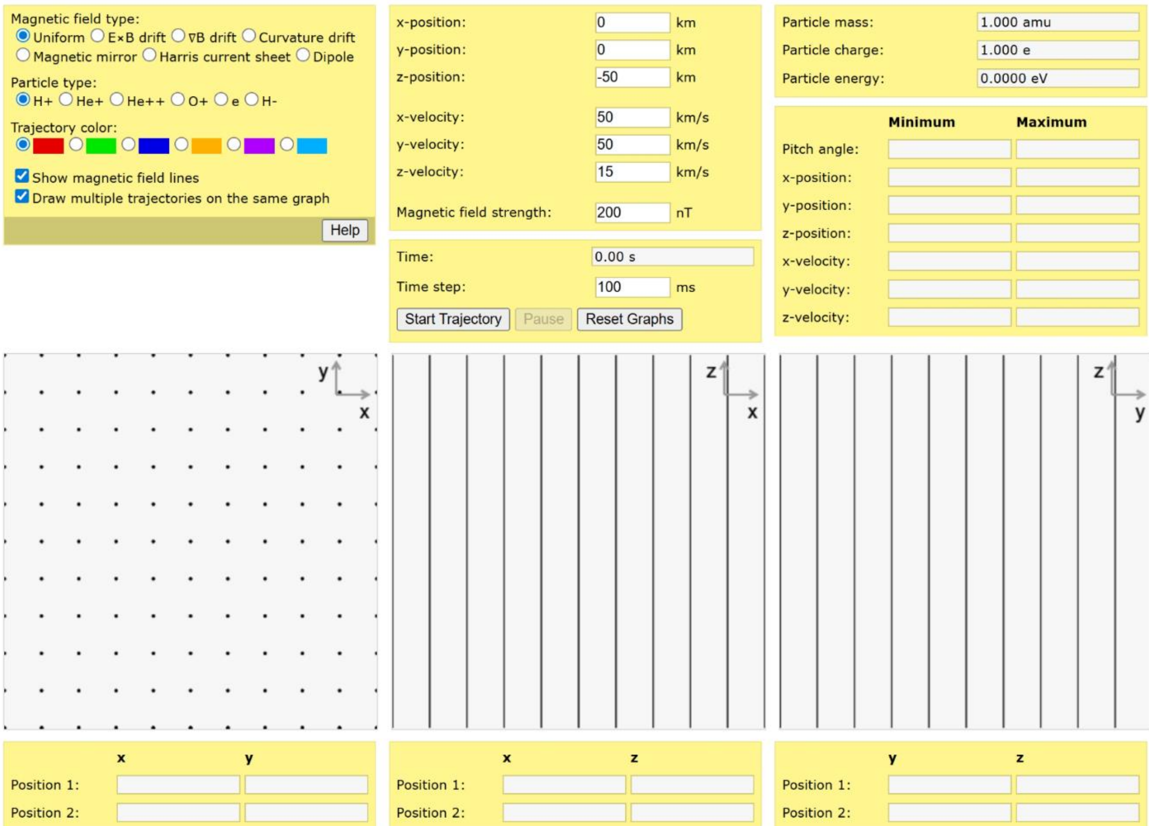



**Figure 11:** The **Particle Motion** module.

This module allows you to follow the three-dimensional trajectories of charged particles through different magnetic field configurations. The equation of motion for each particle is solved numerically, using the Runge-Kutta integration method.

The mass of the electron used in these exercises is the geometric mean of the proton and electron masses. If the true electron mass were used, we would not be able to display the electron and proton gyro motion on the same plot.

The three plots in this module show different orthogonal views of the field configuration. Each plot is 100 km per side (-50 km to +50 km).

Text boxes allow you to specify the initial positions (km) and velocities (km/s) of the selected particle. The magnetic field is along the positive z-direction (i.e., out of the screen in the leftmost plot and upward on the two plots to the right).

The buttons near the center of the window allow you to start and stop drawing the particle's trajectory. The time step allows you to slow down the motion for visualization purposes on fast computers. Particle trajectories may be overlaid, which each trajectory being drawn in a different color.

The middle panel on the left shows the particle mass in proton masses, its charge in electron charges, its energy in electron volts, and the time in minutes and seconds since the trajectory began in simulation time (not observer's time). The panel on the right shows the first and last pitch angels (i.e., the angle between the velocity vector and the magnetic field) and the minimum and maximum positions in all these coordinate directions.

Any position may be measured with the mouse. The particle coordinates will appear in the boxes below the panels. The writing of the measured position toggles between box pairs.

**Magnetic field type**

You have the option of plotting particle trajectories through the following types of magnetic fields. Note that changing the magnetic field type causes plotting to stop and the initial conditions to be reset.

Changing the magnetic field or the electric field will erase the plots.

- **Uniform**

  The uniform magnetic field points in the same direction and is the same strength everywhere. You can adjust the strength by entering a value in the **Field strength** box. The default field strength is 100 nT, and lies along the positive z-axis.

- **E×B drift**

  This option applies an electric field with components across and along the magnetic field. An electric field causes a particle to accelerate parallel to the electric field if it is positively charged and opposite the electric field if it is negatively charged. If the charged particle is moving in a magnetic field, the component of the electric field along the magnetic field causes the particle to accelerate.

  The component of the electric field across the magnetic field leads to drift motions perpendicular to the magnetic field because the particle gyro radius alternatively increases and decreases as it is accelerated and decelerated by the electric field. Use the text boxes to specify the magnetic and electric field strengths and the angle of the electric field to the magnetic field.

  In the three plots, the electric field is directed along the positive x-axis for an angle of 90°. For 0° it is aligned along the positive z-axis, i.e., parallel to the magnetic field.

- **∇B drift**

  Gradients in magnetic fields cause particles to drift perpendicular to the magnetic field and the gradient if the particles have energy perpendicular to the magnetic field. In this option, the magnetic field lines are still straight; i.e., they point in the same direction everywhere, but the field strength is not uniform. We assume the field lines lie along the positive z-direction and field strength increases linearly in the positive x-direction. The field equations are:

  $B_x = 0$

  $B_y = 0$

  $B_z = B_0 + dB_z/dx * (x + 50)$

  where $B_0$ is the field strength at the left border of the xy-plot (x = -50 km; the right border is at +50 km), and $dB_z/dx$ is the gradient of the field strength.

By default, $B_0$ = 10 nT, $d(B_z)/dx$ = 10 nT/10 km, and the resulting field strength on the right-hand side of the plots is 110 nT.

- **Curvature drift**

  Curvature of magnetic field lines causes charged particles to drift if they have energy parallel to the magnetic field. The magnetic field lines in the model are circles of equal field strength in the xy-plane. You can adjust the field strength using the **Field strength** text box, and can move the center of the circles along the x-axis using the **Horizontal center** text box. The field equations are:

  $B_x = B_0 * \cos\theta$

  $B_y = B_0 * \sin\theta$

  $B_z = 0$

  where

  $B_0$ is the field strength

  $\theta = \arctan(y / (x - xc))$

  $x_c$ is the position of the center of the circle

  By default, $B_0$ = 100 nT and $x_c$ = 0 km.

- **Magnetic mirror**

  The magnetic mirror configuration, also called the magnetic bottle, is composed of field lines that approach one another near the ends, and diverge in the middle. The stronger fields at the ends can reflect charged particles so that they are trapped in the magnetic field. The mirror ratio is the ratio of the stronger field strength at the end of the bottle to the field at the middle. The field equations are:

  $B_x = B_r * \cos\theta$

  $B_y = B_r * \sin\theta$

  $B_z = B_0 * ((1 - 1 / s) * (1 - f) + 1 / s)$

  where

  s = mirror ratio

  L = bottle scale length

  $f = \mathrm{sech}(z / L)$

  $r = \sqrt{(x^2 + y^2)}$

  $B_r = -B_0 * r * (1 - s) * f * \tanh(z / L) / (2 * L)$

  $\theta = \arctan(y / x)$

  By default, L = 200 km and s = 100. Changing the value of $B_0$ or the mirror ratio, s, will erase the plots.

- **Harris current sheet**

  The Harris current sheet produces a magnetic field that is everywhere along the z-direction, but gradually decreases in strength on the approach to x = 0 with a scale length H.

  The field changes direction at x = 0. This model is often used to describe the plasma sheet in the magnetotail of the Earth. The equations for the field components are:

$B_x = 0$

$B_y = 0$

$B_z = B_0 * \tanh(x / H)$

The default values for $B_0$ and H are 100 nT and 10 km, respectively.

- **Dipole**

  The dipole field configuration is a very important one in naturally occurring plasma. The converging magnetic field lines of a dipole field reflect charged particles and causes them to return to the equator. Thus, the particles can become trapped in this configuration.

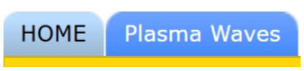
  In addition, the dipole field has curvature and gradients, causing drifts in the particle motion. Near the Earth, this model is a good approximation of the magnetic field. The magnetic field strength used here is 1000 nT at 1000 km from the center, in the equatorial plane.

**Particle type**

You can select to display the trajectories of the following types of particles: H+, He+, He++, O+, e, and H-.

## Plasma Waves

The **Plasma Waves** topic consists of only a single module, also called **Plasma Waves**.




As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let's go over the single module of the **Plasma Waves** topic.

### Plasma Waves

The **Plasma Waves** module allows you to calculate the phase and group velocities of right- and left-handed plasma waves, their ellipticity, wavelengths, and index of refraction with varying magnetic field strength density and wave frequency. This module treats an electron and multiple-ion plasma (up to four species).

The module initially looks like this:

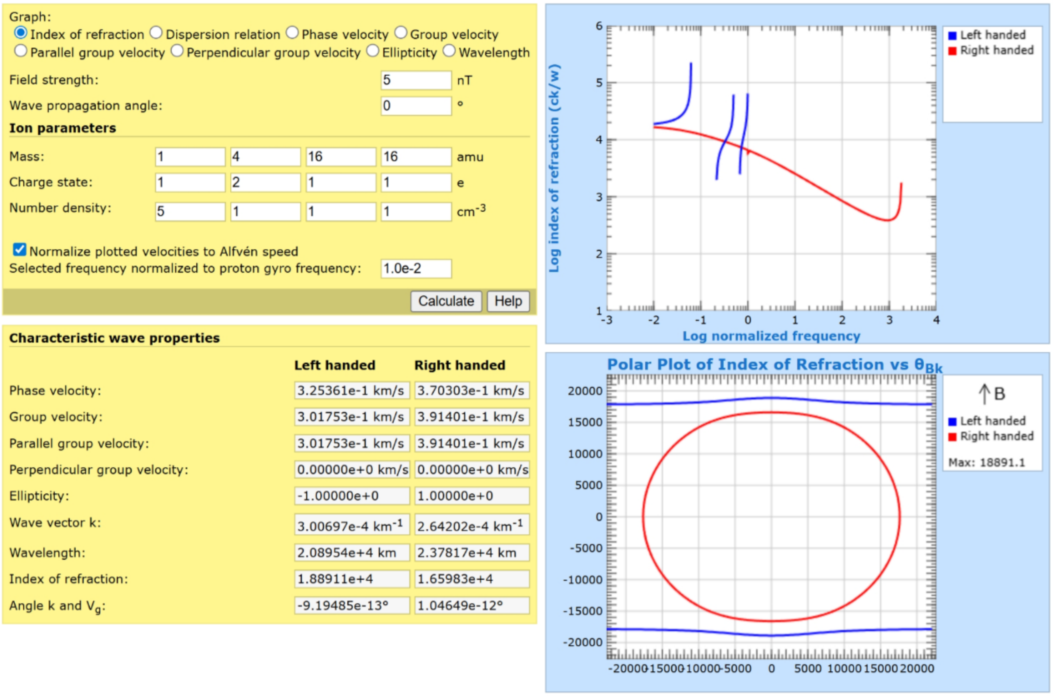



**Figure 12:** The **Plasma Waves** module.

This module calculates various properties (the index of refraction, dispersion relation, phase velocity, group velocity, ellipticity, and wavelength) below the electron cyclotron frequency for linear cold plasma waves in an infinite, homogeneous, and magnetized plasma.

The formulas used are those of T.H. Stix (*The Theory of Plasma Waves*, McGraw-Hill, 1962). The inertial effects of the ions and electrons are retained and all dissipative effects, including collisions, are neglected. This model contains neither sound waves, particle bunching, nor Landau damping, and it does not describe plasma shock waves.

We assume that the plasma consists of multiple ion species, which can be specified by the users, and electrons. The electron density, however, is calculated based on the assumption that the plasma is in a neutral state, i.e., $N_+ = N_-$. The magnetic field B is along the positive z-direction. The wave vector k is in the xz-plane ($k_y = 0$) with an inclination to the magnetic field.

### Graph selection

This module offers the following graph types:

- **Index of refraction**

  This option calculates the index of refraction ($n = c/V_{ph}$ or $ck/\omega$) of the plasma and plots it versus the frequency in the top right panel.

  The bottom right panel displays the polar plot of the index of refraction at the frequency you choose. It shows how the index of refraction at the frequency depends on the propagation angle, the angle between k and B. In this plot, the magnetic field

B is parallel to the vertical axis. Different colors represent the two different modes: blue for left-handed and red for right-handed.

- **Dispersion relation**

  This option shows the dispersion relation of the plasma, i.e., a plot of the frequency of the wave versus its wave number. Note that the frequency scale is vertical.

  The lower graph displays the polar plot of k (1/km) for waves at the frequency you choose. It shows how the value of k at this frequency depends on the propagation angle, the angle between k and B. In this plot, the magnetic field B is parallel to the vertical axis.

- **Phase velocity**

  This option calculates the phase velocity and plots it versus the frequency in the top right panel.

  The lower graph displays the Clemmow-Mullaly-Allis (CMA) diagram for waves at the frequency you choose. It shows how the phase velocities of waves at this frequency depend on the propagation angle, the angle between k and B. In this plot, the magnetic field B is parallel to the vertical axis.

- **Group velocity**

  This option calculates the group velocity and plots it versus the frequency in the top right panel.

  The lower graph displays the polar plot of the group velocity of the wave at the frequency you choose. It shows how the direction of group velocities is controlled by the magnetic field direction. In this plot, the magnetic field B is parallel to the vertical axis. The polar plot displays the group velocity as a function of the B-$V_{gr}$ angle.

  Note that this polar plot does not indicate the direction of wave vector k but the direction of the group velocity. In general, the group velocity is not parallel to the phase velocity.

- **Parallel group velocity**

  This option calculates the group velocity parallel to the wave vector k and plots it versus the frequency in the top right panel.

  The lower graph displays the polar plot of parallel group velocity for waves at the frequency you choose. It shows how the parallel group velocity of the wave at this frequency depends on the propagation angle, the angle between k and B. In this plot, the magnetic field B is parallel to the vertical axis.

  Note that the parallel group velocity refers to the component of group velocity along the wave vector k, or phase velocity, but not the magnetic field B.

- **Perpendicular group velocity**

  This option calculates the group velocities perpendicular to the wave vector k and plots it versus the frequency in the top right panel.

  The lower graph displays the polar plot of the perpendicular group velocity of the wave in km at the frequency you choose. It shows how the perpendicular group velocity of the wave at this frequency depends on the propagation angle, the angle between k and B. In this plot, the magnetic field B is parallel to the vertical axis.

Note that the perpendicular group velocity refers to the component of group velocity transverse to the wave vector k or phase velocity, but not the magnetic field B.

- **Ellipticity**

  This option calculates the wave ellipticity and plots it versus the frequency in the top right panel.

  The wave ellipticity is defined as $iB_y/B_{xz}$, where $B_y$ is the magnetic field perturbation in the y-direction and $B_{xz}$ is the magnetic perturbation in the xz-plane. In other words, $B_y$ is perpendicular to the background magnetic field B and thus is the purely transverse component. $B_{xz}$ is in the plane containing both the k vector and the background magnetic field, and thus it contains both transverse and compressional components. Positive ellipticity represents right-handed polarized wave and negative ellipticity represents left-handed polarized wave.

  The lower graph displays the average normal surface for waves at the frequency you choose. It shows how the phase velocities of waves at this frequency depend on the propagation angle, the angle between k and B. In this plot, the magnetic field B is parallel to the vertical axis.

- **Wavelength**

  This option calculates the wavelength and plots it versus the frequency in the top right panel.

  The lower right graph displays the polar plot of the wavelength in km at the frequency you choose. It shows how the wavelength of the wave at this frequency depends on the propagation angle, the angle between k and B. In this plot, the magnetic field B is parallel to the vertical axis.

**Basic parameters**

You can specify the field strength, wave propagation angle, and plasma properties (mass, charge states, and number density) of at most four ion species by entering those values in the text boxes.

**Normalization**

The **Normalize the quantities plotted** check box allows you to graph normalized values, as explained below.

- Select this option to normalize the frequency f by the cyclotron frequency of the ion species in the first column ($f_{ci1}$); the wavenumber k is normalized by $f_{ci1}/V_A$, where $V_A$ is the Alfvén velocity based on the total mass density; and the speed v is normalized by the Alfvén velocity $V_A$, the velocity of the left-hand mode parallel to the magnetic field at very low frequencies.
- Clear this option to plot the original values. The frequency f is in Hz; the wavenumber k is in 1/km, the speed v is in km/s, and the index of refraction is dimensionless.

### Characteristic wave properties

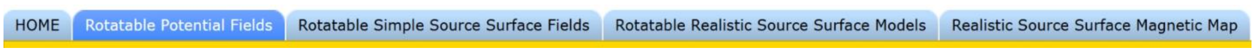
If you want to have a better understanding of the wave at one particular frequency, you can select a frequency by either clicking in the top right panel or manually entering the frequency value under the **Selected frequency** text box to the left.

If you click in the top right panel, the chosen frequency will be marked as a vertical dashed line (except in the dispersion relation graph, in which case it will be marked as a horizontal dashed line).

The bottom left panel displays characteristic wave properties at the frequency you choose. The two columns give the properties for each of the two modes whenever either mode propagates at the chosen frequency. The lower right panel shows the corresponding wave properties.

## POTENTIAL FIELD

The **Potential Field** topic consists of four modules, **Rotatable Potential Fields**, **Rotatable Simple Source Surface Fields**, **Rotatable Realistic Source Surface Models**, and **Realistic Source Surface Magnetic Map**. You can switch between these four modules using the row of tabs along the top of the page:




As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

Let's go over the four modules of the **Potential Field** topic.

### ROTATABLE POTENTIAL FIELDS

The **Rotatable Potential Fields** module allows you to explore the magnetic field configurations of multipolar magnetic fields of different order and degree.

The module initially looks like this:

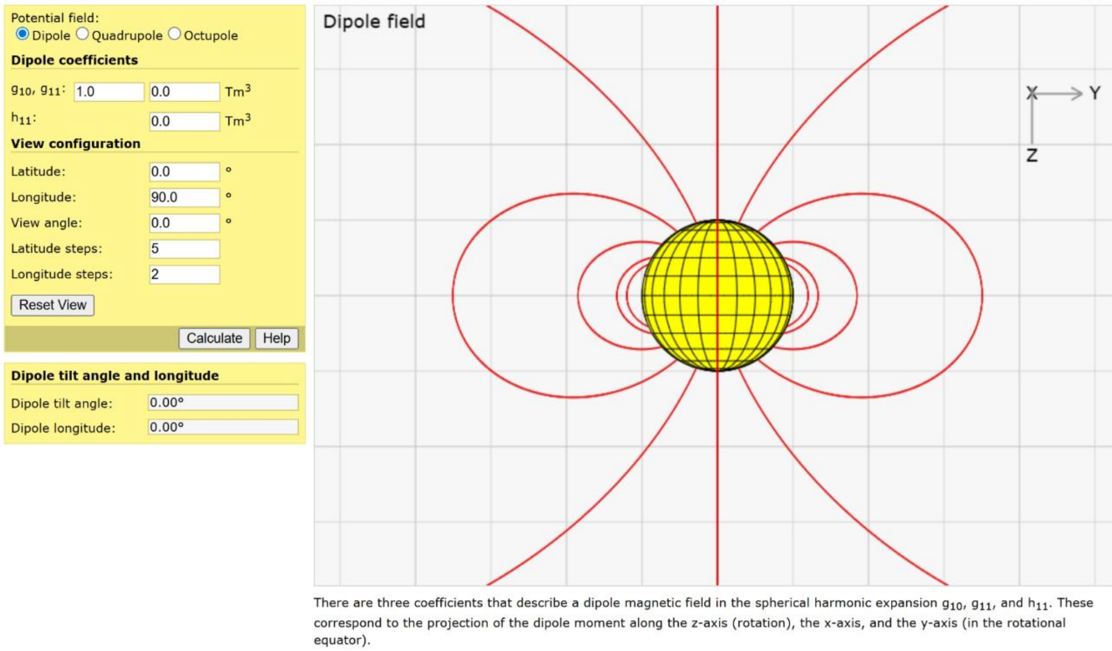



**Figure 13:** The **Rotatable Potential Fields** module.

This module allows you to specify the coefficients that describe a dipole, quadrupole, or octupole magnetic field in a spherical harmonic expansion and to visualize the resulting magnetic field.

## SPHERICAL HARMONIC COEFFICIENTS OF THE INTERNAL FIELD

The internal magnetic field is the field that is generated from the planet's interior and in contrast to the external magnetic field, which is associated with the current system outside the planet. So, outside of the region where the internal magnetic field is generated, it can be represented as the gradient of a scalar potential.

$$B_{internal} = -\nabla V$$

The potential, V, can be expressed by the sum of spherical harmonics through the following expression:

$$V = a\ \Sigma_{n=1-max}\ \Sigma_{m=0-n}\ (a/r)^{n+1}\ (g(n, m)\cos(m\varphi) + h(n, m)\sin(m\varphi))\ P(n, m, \cos\theta)$$

The expression for the magnetic field using the previous two equations, in spherical coordinates, is as follows:

$$B_r = \Sigma_{n=1-max}\ \Sigma_{m=0-n}\ (n+1)\ (a/r)^{n+2}\ (g(n, m)\cos(m\varphi) + h(n, m)\sin(m\varphi))\ P(n, m, \cos\theta)$$

$$B_\theta = \Sigma_{n=1-max} \Sigma_{m=0-n} (a/r)^{n+2} (g(n, m) \cos(m\varphi) + h(n, m) \sin(m\varphi))$$

$$dP(n, m, \cos\theta) / d\theta$$

$$B_\varphi = (1/\sin\theta) \Sigma_{n=1-max} \Sigma_{m=0-n} m (a/r)^{n+2} (g(n, m) \sin(m\varphi) - h(n, m) \cos(m\varphi))$$

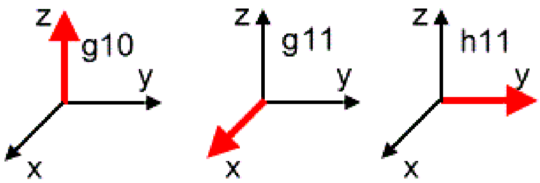
$$P(n, m, \cos\theta)$$

where a is the equatorial radius of the Sun; r is the radial distance to the Sun's center from the point of field evaluation, and angles θ and φ are colatitudes and longitude, respectively. P(n, m, cos θ) are Schmidt-normalized associated Legendre functions of degree n and order m, and g and h are the internal field parameters (Schmidt coefficients).

This model is a magnetic multipole representation of the internal field. This module uses the previous three equations to plot the magnetic field lines.

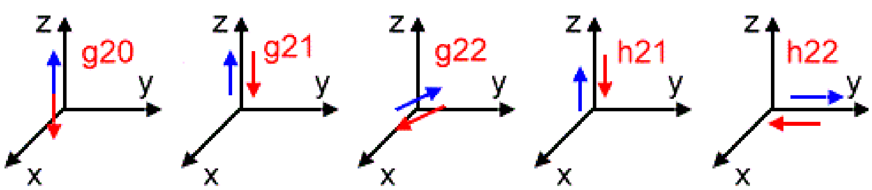
**Dipole coefficients**

n = 1 in the above equations.



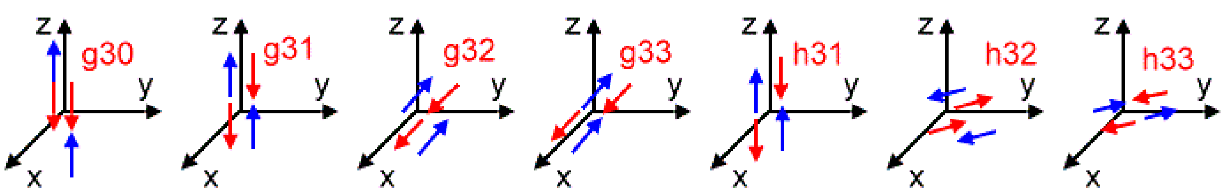

**Quadrupole coefficients**

n = 2 in the above equations.




**Octupole coefficients**

n = 3 in the above equations.




### MAGNETIC MULTIPOLE CONFIGURATION

The definition for the magnetic/electric multipole is that the nth multipole is composed of two opposite (n-1)th multipoles separated by a small distance dl. In the real world, the base magnetic multipole is a dipole, which by convenient comparison we think are composed of two "magnetic monopoles" separating by dl1. From that, any high-order multipoles can be assembled by using the above definition.

Spherical harmonic expansion decompose a magnetic scalar potential to a multipole forms starting from the magnetic dipole. For the nth multipole, it uses 2n+1 coefficients to describe the different forms of this multipole. The spherical harmonic coefficients are ordered as: $g_{10}$, $g_{11}$, $h_{11}$, $g_{20}$, $g_{21}$, $g_{22}$, $h_{21}$, $h_{22}$, etc.

## MAGNETIC DIPOLE

There is only one separating vector dl1 for dipole, thus dipole is always linear. While magnetic monopoles do not exist in isolation, it is useful to think of magnetic dipole in analogy to electric dipole with separated charges along some axis.

There are three coefficients that describe a dipole magnetic field in the spherical harmonic expansion, $g_{10}$, $g_{11}$, and $h_{11}$. These correspond to the projection of the dipole moment along the z-axis (rotation), the y-axis and the x-axis (in the rotational equator) respectively.

In this module, you can specify these three components and visualize the resulting magnetic field, by clicking **Calculate**. The module also allows you to view the magnetic field at different latitudes and longitudes. The module also displays the dipole tilt angle and dipole longitude at bottom left.

## MAGNETIC QUADRUPOLE

A quadrupole is composed of two anti-dipoles separating by a distance dl2. So the initial thoughts would be that we need six coefficients to describe them (dl1 and dl2).

In a linear type quadrupole dl2 is parallel to dl1, whereas in a planar type quadrupole dl2 is perpendicular to dl1. However, a planar type quadrupole can be composed of two linear quadrupoles by perpendicularly placing them, this relationship decreases one degree of freedom, so only five coefficients are needed. That these five configurations provide all possible quadrupole fields (separated dipole pairs) can be shown by expressing the dipoles as separated monopoles and comparing.

The five coefficients that describe a quadrupole magnetic field in a spherical harmonic expansion are:

$g_{20}$: linear type quadrupole, along z-axis, corresponds to separation along the z-axis (rotational). This is called zonal (rotationally symmetric).

$g_{21}$: planar type quadrupole, in the y-z plane, $g_{21}$ has the dipole moment along the z-axis and separated in the y-axis.

$h_{21}$: planar type quadrupole, in the x-z plane, $h_{21}$ has the dipole moment along z-axis and separated in the x-axis.

$g_{22}$: planar type quadrupole, in the x-y plane, $g_{22}$ has the dipole moment along a direction at 45° to the x- and y-axis and a separation along a line at 45° to the x- and y-directions.

$h_{22}$: planar type quadrupole, in the x-y plane, $h_{22}$ has the dipole moment along y-axis and separated in the x-axis.

In this module you can specify these five components and visualize the resulting magnetic field, by clicking **Calculate**. The module also allows you to view the magnetic field at different latitudes and longitudes. The module also allows you the flexibility of number of lines plotted. At the right bottom, there are two input boxes for longitude and latitude. The number of field lines plotted by the program along longitude and latitude are double the value specified by the user in these boxes.

For best viewing of the result you would need proper view angle and density of lines. Sample values of these variables for good viewing, when only single spherical harmonic is present are provided below.

| **Spherical harmonic** | **Lat.** | **Long.** | **Lat. steps** | **Long. steps** |
|---|---|---|---|---|
| $g_{20}$ | 0° | any | 5 | 5 |
| $g_{21}$ | 0° | 90° | 5 | 5 |
| $h_{21}$ | 0° | 0° | 5 | 5 |
| $g_{22}$ | 90° | 90° | 7 | 7 |
| $h_{22}$ | 90° | 90° | 7 | 7 |

To obtain the best view for a combination of different spherical harmonics, you need to try different orientations and line densities. If you specify a large number of latitude and longitude lines, then the program will take more time to plot the lines. (The time the program takes to plot the field lines is proportional to the density of field lines coefficients for longitude and latitude you have specified).

### MAGNETIC OCTUPOLE

An octupole is composed from two anti-quadrupoles separating by distance dl3. The initial thoughts would be that we need nine coefficients to describe them (dl1, dl2, and dl3).

In linear type octupole dl3 is parallel to dl1 and dl2, in planar type octupole dl3 is perpendicular to dl1 or dl2, and they are all in one plane, and in cubic type octupole dl3, dl1, and dl2 are perpendicular to each other. However, a planar type octupole can be composed of two linear octupoles by perpendicularly placing them, and similarly a cubic type octupole can be composed by two planar octupoles, thus reducing the degree of freedom by two, so we need only seven coefficients. That these seven octupole configurations provide all possible octupole fields (separated quadrupole pairs) can be seen by expressing the moments as distributed monopoles and comparing.

The seven coefficients that describe an octupole magnetic field in the spherical harmonic expansion are $g_{30}$, $g_{31}$, $h_{31}$, $g_{32}$, $h_{32}$, $g_{33}$, and $h_{33}$. The $g_{30}$ is a zonal moment created by separating two oppositely-directed zonal quadrupoles along the z-axis (rotational). Two planar octupoles $g_{31}$ and $h_{31}$ are constructed from linear quadrupoles; two planar types $g_{33}$ and $h_{33}$ are constructed from planar quadrupoles and there are two cubic types $g_{32}$ and $h_{32}$.

In this module you can specify these five components and visualize the resulting magnetic field, by clicking **Calculate**. The module also allows you to view the magnetic field at different latitudes and longitudes. The module also allows the user the flexibility of number of lines plotted. At the right bottom, there are two input boxes for longitude and latitude. The number of field lines plotted by the program along longitude and latitude are double the value specified by the user in these boxes.

For best viewing of the result you would need proper view angle and density of lines. Sample values of these variables for good viewing, when only single spherical harmonic is present are provided below.

| **Spherical harmonic** | **Lat.** | **Long.** | **Lat.** | **Long.** |
|---|---|---|---|---|
| $g_{30}$ | 0° | any | 5 | 5 |
| $g_{31}$ | 0° | 90° | 5 | 5 |
| $h_{31}$ | 0° | 0° | 5 | 5 |
| $g_{32}$ | 90° | 90° | 11 | 11 |
| $h_{32}$ | 90° | 90° | 11 | 11 |
| $g_{33}$ | 90° | 90° | 11 | 11 |
| $h_{33}$ | 90° | 90° | 11 | 11 |

To obtain the best view for a combination of different spherical harmonics, you need to try with different values of rotation and density of lines, to obtain best view. If you provide a very large value for density of lines along latitude and longitude, then the program will take more time to do plotting. (The time the program takes to plot the field lines is proportional to the density of field lines coefficients for longitude and latitude the user has provided).

### ROTATABLE SIMPLE SOURCE SURFACE FIELDS

The **Rotatable Simple Source Surface Fields** module allows you to examine the effect of the distance of the source surface from the Sun on the source region of the solar wind from multipole magnetic fields.

The module initially looks like this:

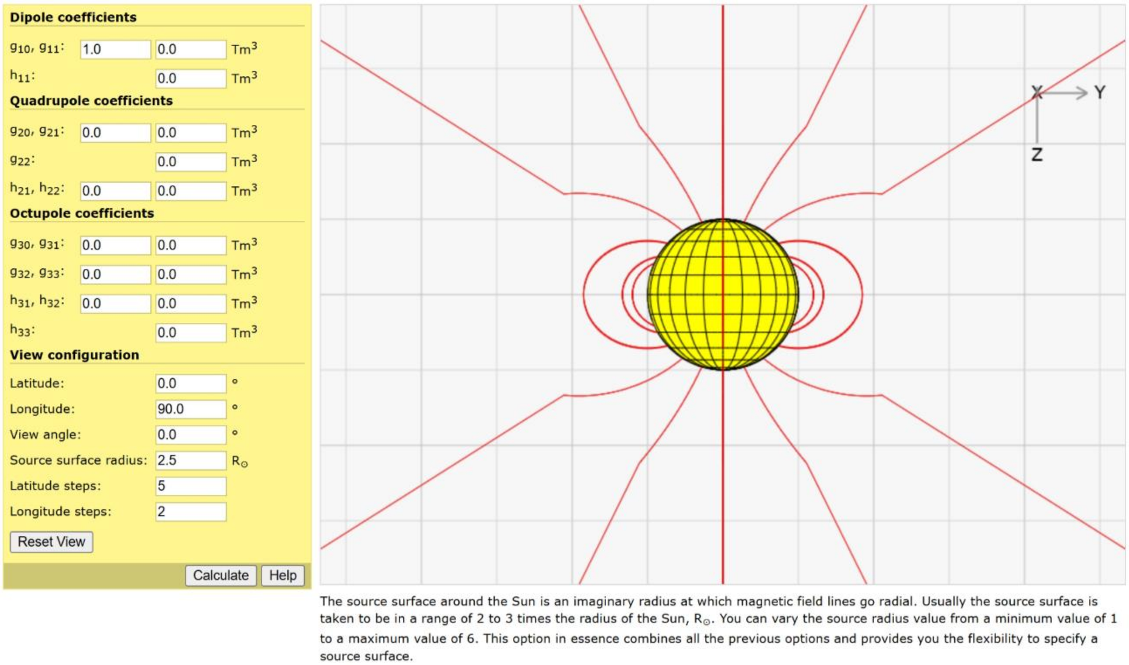



**Figure 14:** The **Rotatable Simple Source Surface Fields** module.

For introductory comments and definitions of the dipole, quadrupole, and octupole coefficients, see the description of the **Rotatable Potential Fields** module above.

## ROTATABLE REALISTIC SOURCE SURFACE MODELS

The **Rotatable Realistic Source Surface Models** module allows you to examine the magnetic topology of realistic potential field models.

The module initially looks like this:

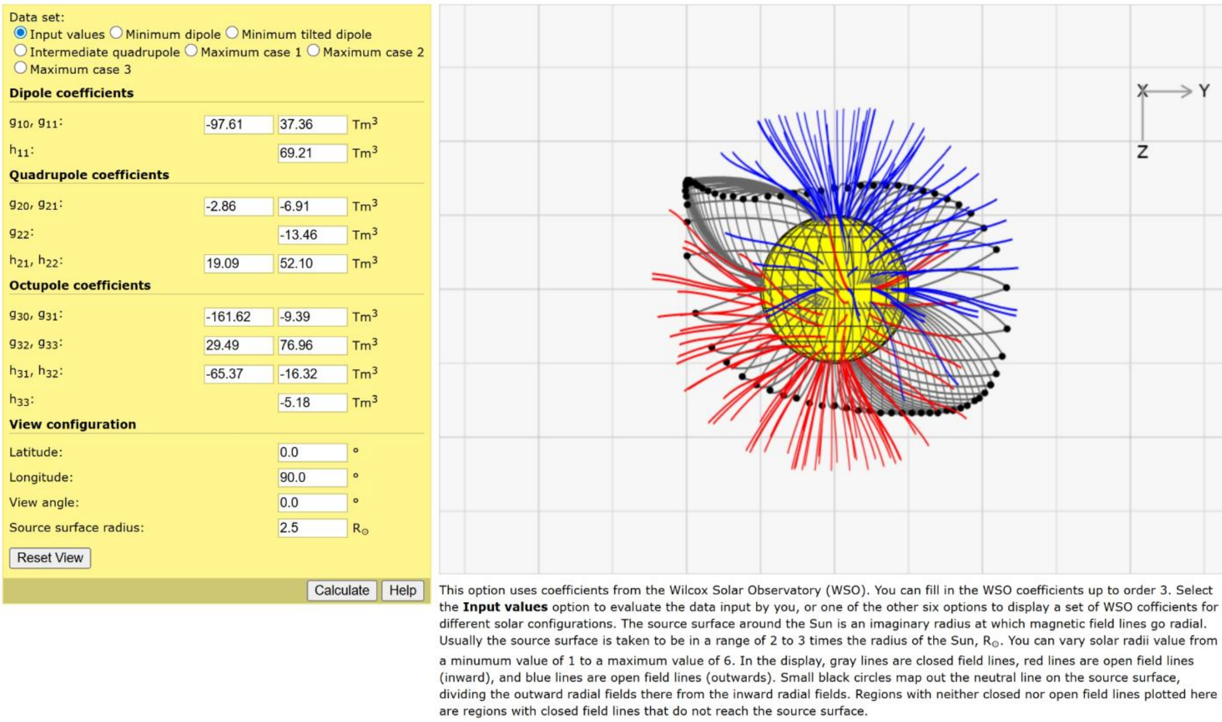



**Figure 15:** The **Rotatable Realistic Source Surface Models** module.

For introductory comments and definitions of the dipole, quadrupole, and octupole coefficients, see the description of the **Rotatable Potential Fields** module above.

## REALISTIC SOURCE SURFACE MAGNETIC MAP

This module allows you to examine maps of the open and closed field regions of the Sun for realistic potential field models.

The module initially looks like this:

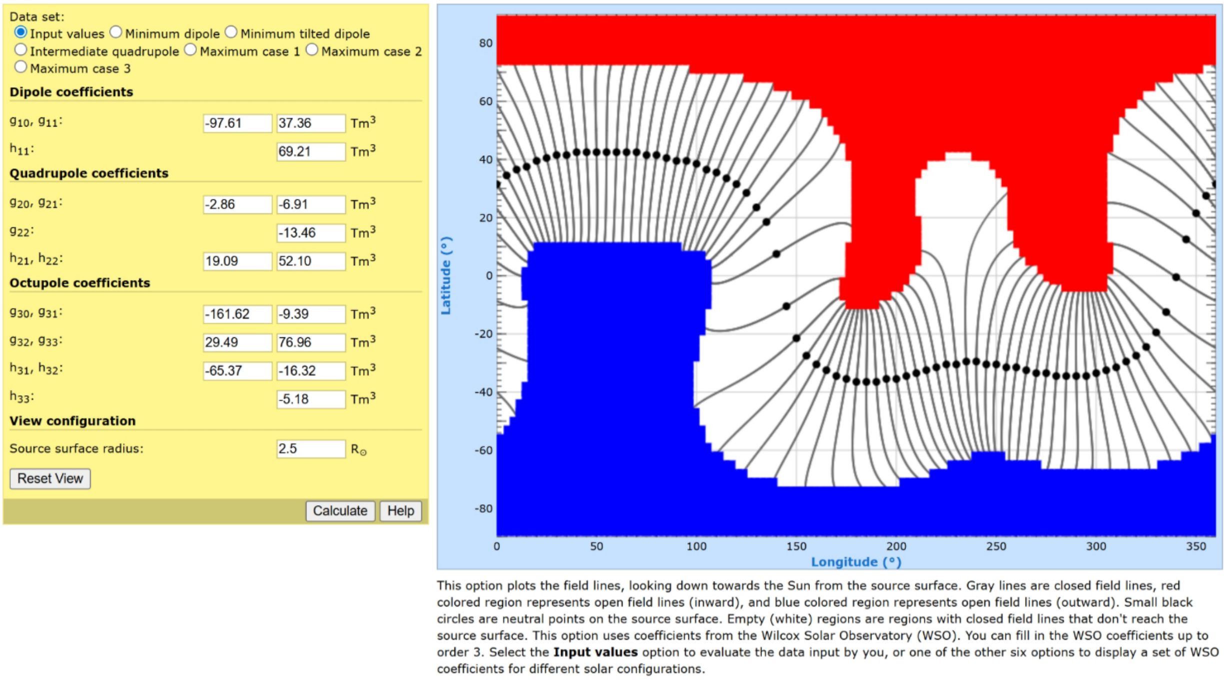



**Figure 16:** The **Realistic Source Surface Magnetic Map** module.

For introductory comments and definitions of the dipole, quadrupole, and octupole coefficients, see the description of the **Rotatable Potential Fields** module above.

## Solar Wind

The **Solar Wind** topic consists of three modules, **Parker Spiral Model**, **Neutral Sheet Model, 3-D Structure**, and **Neutral Sheet Model, Stream Interactions**. You can switch between these three modules using the row of tabs along the top of the page:


As mentioned above, you can always click the **HOME** tab to get back to the Space Physics Exercises home page.

The *solar wind* is the expanding outer atmosphere of the Sun that fills interplanetary space. It is a primarily hydrogen plasma that flows approximately radially from the region in the chromosphere or corona where it is accelerated. Typical solar wind speeds are around 400 km/s, although speeds as low as 250 km/s and over 1000 km/s have been observed. The speed does not vary with heliocentric distance r but the density of particles falls off as $1/r^2$ due to the radial expansion. The solar wind at 1 AU has a density of about 10 particles per $cm^3$.

The existence of the solar wind was predicted by E.N. Parker in the late 1950s. Parker also predicted that the highly-conducting solar-wind plasma would carry the Sun's magnetic field with it into interplanetary space. Parker showed that this interplanetary magnetic field (IMF) should have a spiral geometry due to the combination of radial outflow and solar rotation. His model for the IMF has since been called the Parker spiral model.

Let's go over the three modules of the **Solar Wind** topic.

## Parker Spiral Model

The **Parker Spiral Model** module allows you to examine the dependence of the Parker spiral model on parameters such as velocity.

The module initially looks like this:

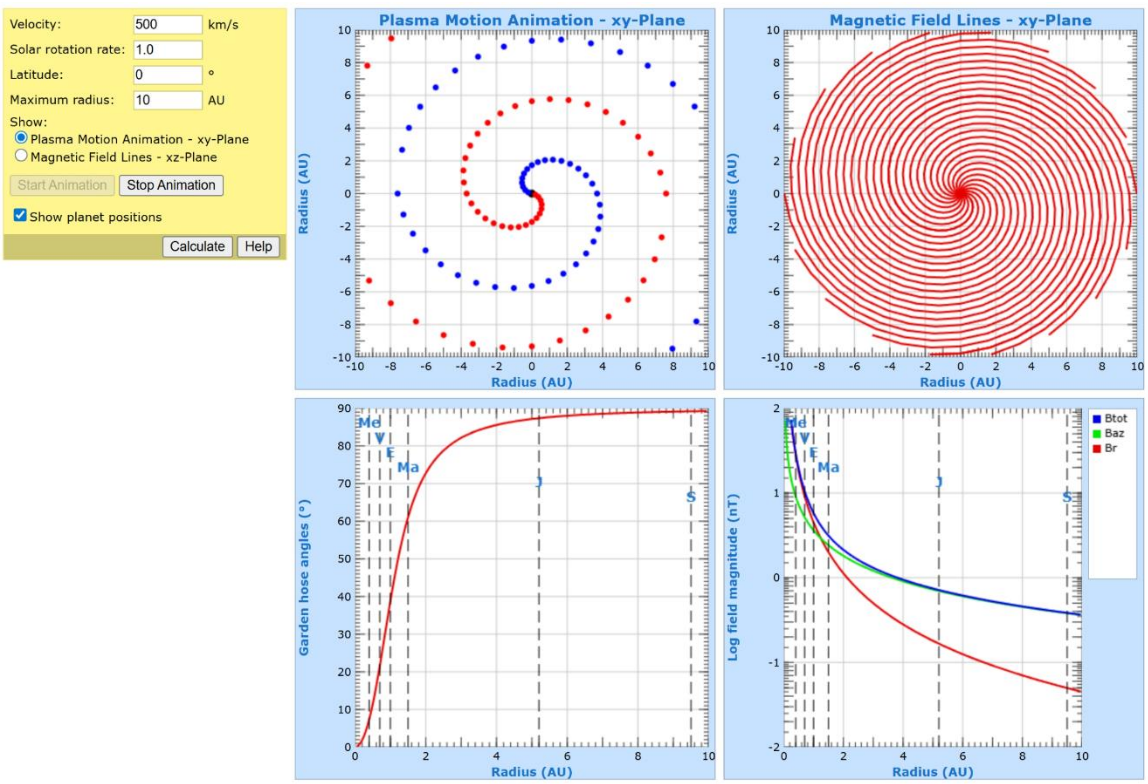



**Figure 17:** The **Parker Spiral Model** module.

The Parker spiral concept is based on the idea that the solar wind stretches the solar magnetic field out into space. It would be essentially radial in the absence of solar rotation. However, because the Sun is rotating (at a speed of about once a month as seen from Earth) the field winds up into the shape of an Archimedean spiral. In the solar equatorial plane, the angle a that the IMF makes with respect to the radial is given by

$$a = \arctan(r\omega / V)$$

where ω is the angular rotation rate of the Sun and V is the solar wind speed. This angle is known as the "garden hose" angle because of the similarity in appearance of the IMF lines and the pattern of water jets produced by a rotating garden sprinkler. At 1 AU the average garden hose angle is about 45°.

## Graph Options

The graph in the upper-right corner as well as the two lower graphs are fixed, but you can choose which of two graphs to see displayed in the upper-left corner:

- **Plasma Motion Animation - xy-Plane**

  When you chose this option, the upper-left graph shows elements of plasma emitted radially from the Sun from two regions on opposite sides of the Sun. As the Sun rotates (about once every 27 days) the radially moving plasma elements line up in a spiral pattern. If these radially moving plasma elements carried the magnetic field of the "active" regions with them, the magnetic field lines would form spirals as on the upper-right graph.

- **Magnetic Field Lines - xz-Plane**

  When you chose this option, you can examine how the model field lines behave out of the equatorial plane. In three dimensions, the Parker spiral field is described by the equations

  $B_r = B_0 (r_0 / r)^2$

  $B_\theta = 0.0$

  $B_{azi} = B_0 r_0^2 \omega \sin(\text{latitude}) / rV$

  Here $r_0$ is the radius of some "source surface" at which the field is radial and has magnitude $B_0$. $\omega$ is the rotation rate of the Sun, r is the radial distance of the observer, and V is the solar wind velocity. The spherical coordinate system is referenced to the solar rotation axis. The three orthogonal directions of the field are: radial, $B_r$; colatitude, $B_\theta$; and the (clockwise) azimuthal, $B_{azi}$.

  The top two graphs show the projections of the spiraling magnetic field lines along cones of constant latitude, projected into two orthogonal planes: that containing the solar rotational axis (left) and that into the rotational equatorial plane (right). The spiral angle of the magnetic field can be adjusted by changing the solar wind velocity and the rotation rate of the Sun.

  Note that in reality the rotation rate features on the solar surface do vary with latitude. A value of unity here is equivalent to the normal equatorial rotation rate of 27 days as seen from the Earth.

The bottom two graphs show the radial variation of the angle between the magnetic field and the radial direction (garden hose or spiral angle) and the radial variation of the magnetic field strength ($B_{tot}$), the azimuthal field ($B_{az}$), and the radial field ($B_r$) in nanoteslas.

You can show or hide the positions of the planets on the bottom graphs by setting or clearing the Show planet positions check box. The positions of the planets are indicated on the graphs by dashed vertical lines labeled Mercury, Venus, Mars, Earth, Jupiter, Saturn, Uranus, Neptune, and Pluto.

## Neutral Sheet Model, 3-D Structure

The **Neutral Sheet Model, 3-D Structure** module allows you to experiment with different influences on current sheet topology, including magnetic moment tilt angle (with respect to the solar rotation axis), relative contribution of quadrupole and dipole moments, and solar wind velocity.

The module initially looks like this:

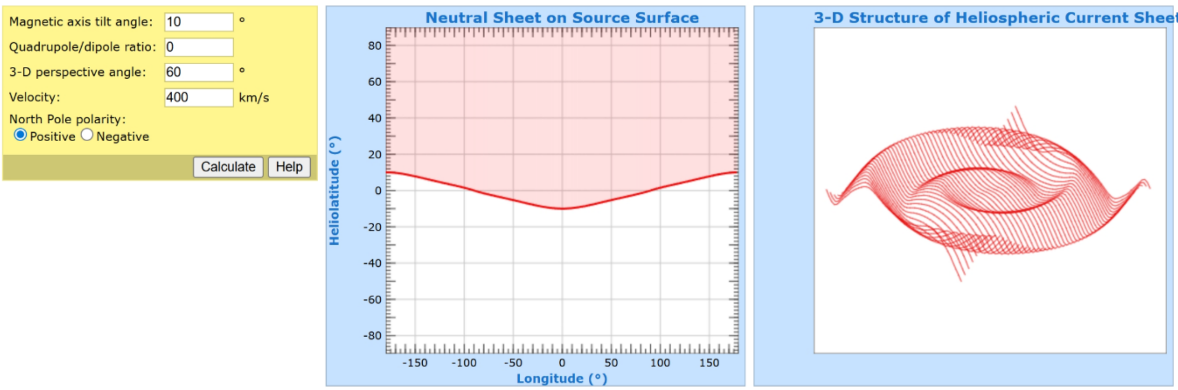



**Figure 18:** The **Neutral Sheet Model, 3-D Structure** module.

## Heliospheric Current Sheet

If the solar magnetic field is originally dipolar before the solar wind stretches it into space, and the dipole axis is aligned with the solar rotation axis, one would expect the interplanetary magnetic field (IMF) to show different "polarities" in the northern and southern heliosphere. These oppositely directed "toward" (outward) and "away" (inward) fields should be separated by a planar current sheet in the equatorial plane.

Observations at 1 AU in the early 1970s showed that the IMF usually exhibits alternating polarities during the course of each 27-day apparent solar rotation. Sometimes there are two "sectors" per rotation, but there are often four. Several researchers (including M. Schulz and H. Alfvén) recognized that a dipole axis tilted at an angle with respect to the solar rotation axis would produce the two-sector pattern, while the four-sector pattern would be created by the addition of a solar quadrupole moment. The appearance of the heliospheric current sheet in such circumstances invoked the "ballerina skirt" analogy.

The graph on the left shows the solar surface mapped onto a Cartesian grid of latitude (vertically) and longitude (horizontally) measured versus the rotational equator and pole. Shading indicates the polarity of the magnetic field crossing the surface. The graph on the right shows a perspective view of the current sheet separating radially outward fields from radially inward fields.

This module allows you to control the tilt of the magnetic dipole axis, the amount of quadrupole field present, the viewing perspective, the velocity of the solar wind, and the polarity of the field. Note that when the tilt angle is zero, the rotational and magnetic dipole axes are aligned.

The added quadrupole moment produces four alternately directed "orange slice" regions of fields around the Sun. This moment is equivalent to the close positioning of two oppositely-directed dipoles, each in the equatorial plane separated along their lengths. The addition of this quadrupole moment to the dipole field warps the current sheet so that the region of net outward field and of net inward field penetrates above and below the rotational equator every 180° around the equator.

## Neutral Sheet Model, Stream Interactions

The **Neutral Sheet Model, Stream Interactions** module allows you to experiment with a phenomenological model of stream interactions based on a parameterized treatment of the physics, and to "simulate" plasma and field data that might be observed by spacecraft at various heliocentric distances.

The module initially looks like this:

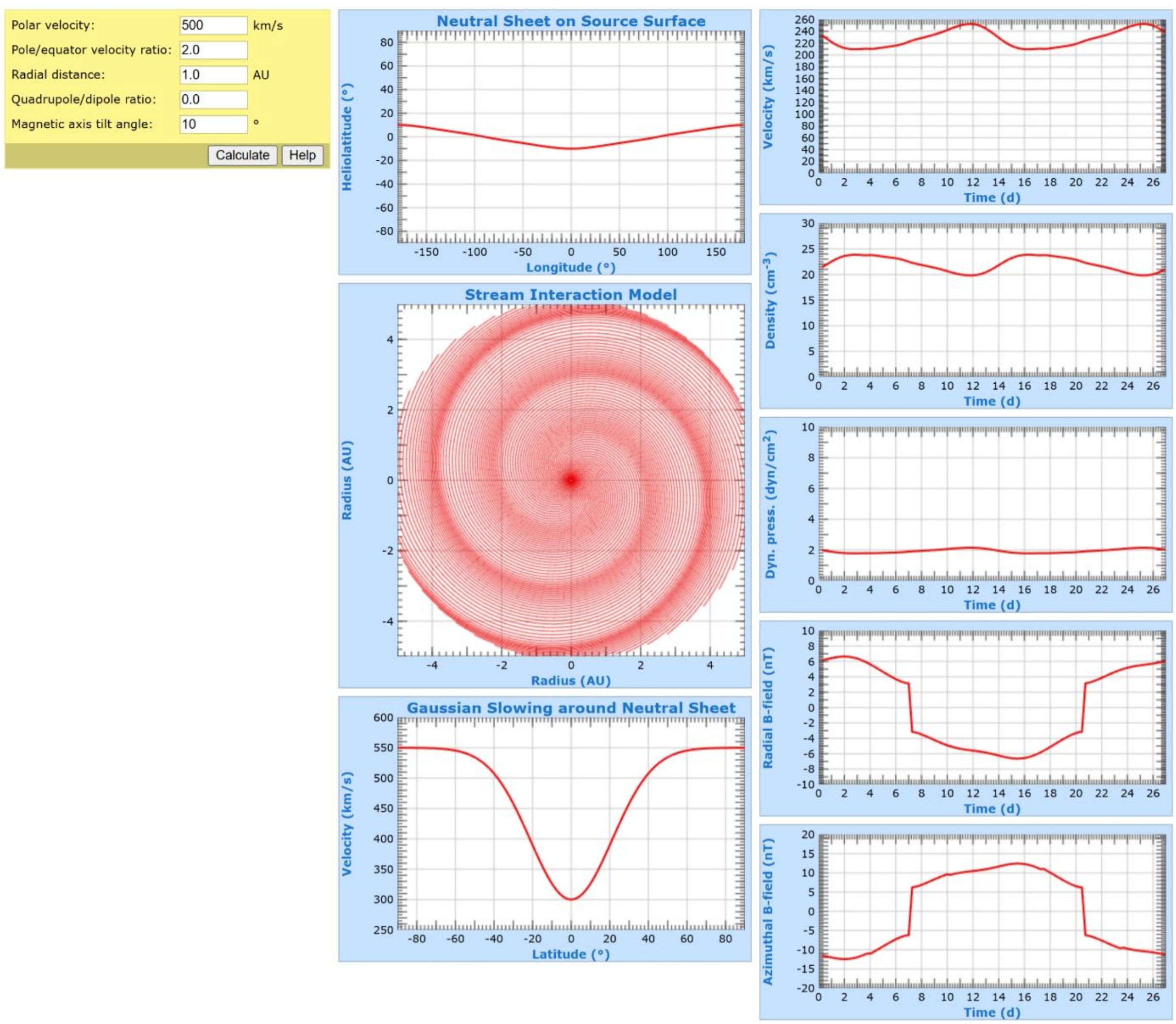



**Figure 19:** The **Neutral Sheet Model, Stream Interactions** module.

This module allows you to experiment with a phenomenological model of stream interactions based on a parameterized treatment of the physics (described by Hakamada, K., and S.-I. Akasofu, "Simulation of Three-Dimensional Solar Wind Disturbances and Resulting Geomagnetic Storms," *Space Sci. Rev.*, 31, 3-70, 1982), and to "simulate" plasma and field data that might be observed by spacecraft at various heliocentric distances.

## STREAM INTERACTIONS MODEL

Spacecraft measurements show that the solar wind velocity depends on the distance from the heliospheric current sheet. The velocity can be 2 to 3 times as high near the magnetic "poles" of the Sun (the location of polar coronal holes) as it is near the current sheet. This "heliomagnetic latitude" dependence produces a high speed/low speed "stream" structure that is especially prominent at low heliographic latitudes when the dipole tilt is significant or a strong quadrupole contribution to the Sun's field is present.

When there is a non-zero dipole tilt or finite quadrupole moment the solar rotation brings different source velocities to the base of a given radial direction in the inertial frame. Since the plasma transport is radial, high-speed plasma that is launched behind lower speed plasma will plow into it and create a "stream interaction" region. Due to the fact that the magnetized plasmas do not interpenetrate, the interaction speeds up the slower plasma and slows down the faster plasma. A pressure ridge is thus formed that takes a spiral form for the same reasons as the interplanetary magnetic field (IMF).

The distribution of IMF magnitude also shows the interaction regions since it too is compressed. Collisionless shocks may eventually form as the pressure ridges steepen with increasing heliospheric distance, but this usually happens beyond 1 AU.

The graphs on the left show, from top to bottom:

- The neutral sheet on the Sun's surface where the radial component of the magnetic field is zero
- Magnetic field lines in the rotational equator carried outward like spiral threads in the flow attached on one end to the rotating source region
- The latitudinal profile of the velocity measured along meridians of the rotational equator, but referenced to the magnetic equator (neutral sheet)

The graphs on the right show the predicted 1 AU data versus time in steady state as the Sun rotates every 27 days, from top to bottom:

- Velocity
- Density
- Dynamic pressure
- Radial magnetic field
- Azimuthal magnetic field

The controls on the left allow you to adjust the maximum velocity of the observer (out to 1 AU), the quadrupole to dipole ratio, and the tilt angle of the dipole moment. This later angle is set to zero if the quadrupole moment is non-zero. The slow region around the neutral sheet (magnetic equator) has a fixed Gaussian width and variable depth.

For additional information, see the description of the **Neutral Sheet Model, 3-D Structure** module above.